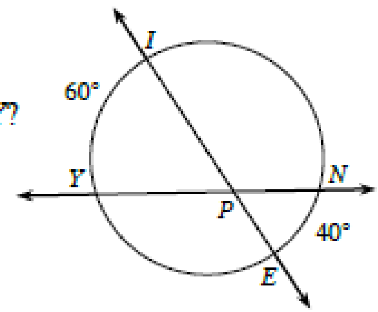
Example 1

In the circle at right, $m\widehat{IY} = 60°$ and $m\widehat{NE} = 40°$. What is $m\angle IPY$?

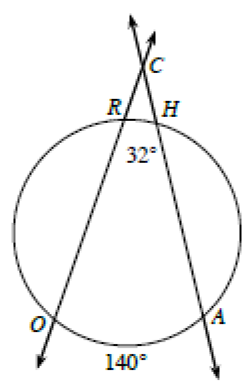


The two lines, $\overline{IE}$ and $\overline{YN}$, are secants since they each intersect the circle at two points. When two secants intersect in the interior of the circle, the measures of the angles formed are each one-half the sum of the measures of the intercepted arcs. Hence $m\angle IPY = \frac{1}{2}(m\widehat{IY} + m\widehat{NE})$ since $\widehat{IY}$ and $\widehat{NE}$ are the intercepted arcs for this angle. Therefore:

$$\begin{aligned} m\angle IPY &= \tfrac{1}{2}(m\widehat{IY} + m\widehat{NE}) \\ &= \tfrac{1}{2}(60° + 40°) \\ &= 50° \end{aligned}$$

Example 2

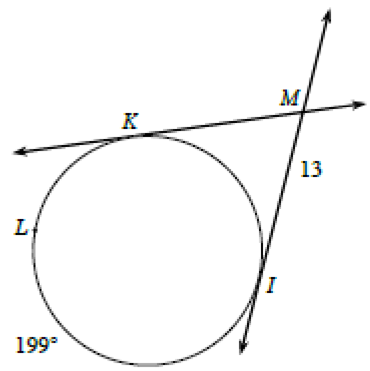
In the circle at right, $m\widehat{OA} = 140°$ and $m\widehat{RH} = 32°$. What is $m\angle OCA$?



This time the secants intersect outside the circle at point C. When this happens, the measure of the angle is one-half the difference of the measures of the intercepted arcs. Therefore:

$$\begin{aligned} m\angle OCA &= \tfrac{1}{2}(m\widehat{OA} - m\widehat{RH}) \\ &= \tfrac{1}{2}(140° - 32°) \\ &= 54° \end{aligned}$$

Example 3



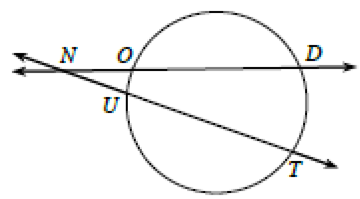
$\overline{MI}$ and $\overline{MK}$ are tangent to the circle. $m\widehat{ILK} = 199°$ and $MI = 13$. Calculate $m\widehat{IK}$, $m\angle IMK$, and the length of $\overline{MK}$.

When tangents intersect a circle we have a similar result as we did with the secants. Here, the measure of the angle is again one-half the difference of the measures of the intercepted arcs. But before we can find the measure of the angle, we first need to find $m\widehat{IK}$. Remember that there are a total of 360° in a circle, and here the circle is broken into just two arcs. If $m\widehat{ILK} = 199°$, then $m\widehat{IK} = 360° - 199° = 161°$. Now we can find $m\angle IMK$ by following the steps shown at right.

$$\begin{aligned} m\angle IMK &= \tfrac{1}{2}(m\widehat{ILK} - m\widehat{IK}) \\ &= \tfrac{1}{2}(199° - 161°) \\ &= 19° \end{aligned}$$

Lastly, when two tangents intersect, the segments from the point of intersection to the point of tangency are congruent. Therefore, $MK = 13$.

Example 4

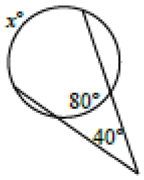


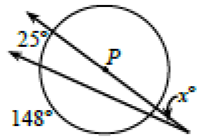
In the figure at right, $DO = 20$, $NO = 6$, and $NU = 8$. Calculate the length of $\overline{UT}$.

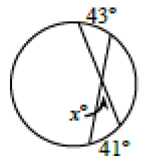
We have already looked at what happens when secants intersect inside the circle. (We did this when we considered the lengths of parts of intersecting chords. The chord was just a portion of the secant. See the Math Notes box in Lesson 10.1.4.) Now we have the secants intersecting outside the circle. When this happens, we can write $NO \cdot ND = NU \cdot NT$. In this example, we do not know the length of $\overline{UT}$, but we do know that $NT = NU + UT$. Therefore we can write and solve the equation at right.

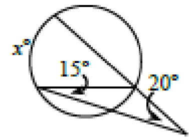
$$\begin{aligned} NO \cdot ND &= NU \cdot NT \\ 6 \cdot (6+20) &= 8 \cdot (8+UT) \\ 156 &= 64 + 8UT \\ 92 &= 8UT \\ UT &= 11.5 \end{aligned}$$

Compute the value of x.

21. 

22. 

23. 

24. 

21. 160 22. 9 23. 42 24. 70