Factor the polynomial and find all its roots.

$$P(x) = x^4 + x^2 - 14x - 48$$

Students learn the Integral Zero Theorem, which says that zeros, or roots of this polynomial, must be factors of the constant term. This means the possible real roots of this polynomial are ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±16, ±24, or ±48. In this case there are 20 possible roots to check! We can check them in a number of ways. One method is to divide the polynomial by the corresponding binomial expression (for instance, if –1 is a root, we divide the polynomial by $(x + 1)$ to see if it is a factor. Another method is to substitute each zero into the polynomial to see which of them, if any, make the polynomial equal to zero. We will still have to divide by the corresponding expression once we have the root, but it will mean less division in the long run.

Substituting values for $P(x)$, we get:

$$\begin{aligned} P(1) &= (1)^4 + (1)^2 - 14(1) - 48 \\ &= 1 + 1 - 14 - 48 \\ &= -60 \end{aligned}$$

$$\begin{aligned} P(-1) &= (-1)^4 + (-1)^2 - 14(-1) - 48 \\ &= 1 + 1 + 14 - 48 \\ &= -32 \end{aligned}$$

$$\begin{aligned} P(2) &= (2)^4 + (2)^2 - 14(2) - 48 \\ &= 16 + 4 - 28 - 48 \\ &= -56 \end{aligned}$$

$$\begin{aligned} P(-2) &= (-2)^4 + (-2)^2 - 14(-2) - 48 \\ &= 16 + 4 + 28 - 48 \\ &= 0 \end{aligned}$$

We can keep going, but we just found a root, $x = -2$. Therefore, $x + 2$ is a factor of the polynomial. Now we can divide the polynomial by this factor to find the other factors.

| | $x^3$ | $-2x^2$ | $5x$ | $-24$ |
|---|---|---|---|---|
| $x$ | $x^4$ | $-2x^3$ | $5x^2$ | $-24x$ |
| $2$ | $2x^3$ | $-4x^2$ | $10x$ | $-48$ |

Factor the polynomials, keeping the factors real.

4. $f(x) = 2x^3 + x^2 - 19x + 36$

5. $g(x) = x^4 - x^3 - 11x^2 - 5x + 4$

4. $f(x) = (x+4)(2x^2 - 7x + 9)$

5. $g(x) = (x+1)(x-4)(x^2 + 2x - 1)$